MATH 119: Midterm 2

Name: Key

Directions:

- Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
- If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
		50

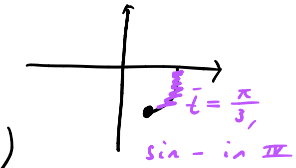
1. Simplify these expressions:

- $3\sin^2\left(-\frac{\pi}{3}\right) + 3\cos\left(\frac{11\pi}{6}\right) + 3\tan(20\pi)$

$\sin\left(-\frac{\pi}{3}\right)$

$= -\sin\left(\frac{\pi}{3}\right)$

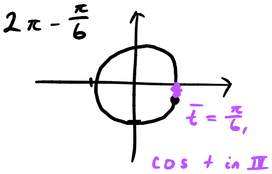
$= -\frac{\sqrt{3}}{2}$



$\cos\left(\frac{11\pi}{6}\right)$ $\quad \frac{11\pi}{6} = 2\pi - \frac{\pi}{6}$

$= \cos\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}$



$\tan(20\pi)$ $\quad 20\pi = 10(2\pi)$

$= \tan(0)$ $\quad$ ↑ end up where you started

$= \frac{0}{1} = 0$

$$= 3\left(-\frac{\sqrt{3}}{2}\right)^2 + 3\cdot\frac{\sqrt{3}}{2} + 3\cdot 0$$

LoE #4 then #5, frac law #1

$$= 3\cdot(-1)^2\cdot\frac{(\sqrt{3})^2}{2^2} + \frac{3\sqrt{3}}{2}$$

$$= 3\cdot\frac{3}{4} + \frac{3\sqrt{3}}{2}$$

$$= \frac{9}{4} + \frac{3\sqrt{3}}{2}\cdot\frac{2}{2}$$

LCD, frac law 1 then 3

$$= \boxed{\frac{9+6\sqrt{3}}{4}}$$

compound fraction, get rid of nested fraction (section 1.4)

https://swrlly.com/teaching/math-118/lectures/1.4

- $\frac{\sec\theta - \cos\theta}{\sin\theta} = \frac{\frac{1}{\cos\theta} - \cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}$

frac law #1

$$= \frac{\left(\frac{1}{\cos\theta} - \cos\theta\right)\cdot\cos\theta}{\sin\theta\cos\theta}$$

dist law

$$= \frac{\frac{1}{\cancel{\cos\theta}}\cancel{\cos\theta} - \cos^2\theta}{\sin\theta\cos\theta}$$

frac law 5

$$= \frac{1-\cos^2\theta}{\sin\theta\cos\theta}$$

$\sin^2\theta + \cos^2\theta = 1$

$$= \frac{\cancel{\sin^2\theta}\ \sin\theta}{\cancel{\sin\theta}\cos\theta}$$

frac law 5

$$= \frac{\sin\theta}{\cos\theta}$$

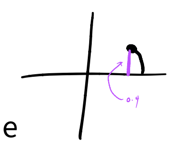
$$= \boxed{\tan\theta}$$

2. Short answer questions.

⚠ Justify each answer with formulas or facts for full credit; do not just write "yes" or "no" ⚠ .

(a) Given $f(x) = \sin(x)$, does there exist $x \in \mathbb{R}$ such that $f(x) = 0.4$? Why or why not?

Yes, x represents the distance and direction you walk along the unit circle. There does exist some distance x where you can walk to get to a y-coordinate of 0.4.



(b) If a mass attached to a spring is moving in simple harmonic motion, can we use the function

$$d(t) = a\sec(\omega t)$$

to model it's displacement? Why or why not?

No, Secant has vertical asymptotes, meaning $\sec(t) \longrightarrow \infty$ as x approaches some value. Simple harmonic motion does not have displacement which grows without bound.

(c) Is it possible for angular speed to be less than linear speed? Why or why not?

Yes, $\underbrace{v}_{\text{linear speed}} = r \cdot \underbrace{\omega}_{\text{angular speed}}$

if $r > 1$ then $\omega < v$.

(d) When proving a trig identity, are we allowed to square both sides? Why or why not?

No; you would be assuming both sides are true.

Moreover you could start with something false and squaring could turn that to a true statement. That would be argument from false premises.

3. Prove these identities:

- $$\frac{(\sin x + \cos x)^2}{\sin x \cos x} = 2 + \sec x \csc x$$

$$LHS = \frac{(\overbrace{\sin x}^{A} + \overbrace{\cos x}^{B})^2}{\sin x \cos x} \underset{\substack{\text{product} \\ (A+B)^2}}{\overset{\text{Special}}{=}} \frac{\sin^2 x + 2 \sin x \cos x + \cos^2 x}{\sin x \cos x}$$

pythagorean identity

$$= \frac{1 + 2 \sin x \cos x}{\sin x \cos x}$$

$$\underset{\text{law 3}}{\overset{\text{undo frac}}{=}} \frac{1}{\sin x \cos x} + \frac{2 \cdot \cancel{\sin x} \cdot \cancel{\cos x}}{\cancel{\sin x}\ \cancel{\cos x}}$$

frac law 5

undo frac law #1

$$= \frac{1}{\sin x} \cdot \frac{1}{\cos x} + 2$$

$$= \csc x \sec x + 2 = RHS$$

- $$\sin\left(\frac{\pi}{2} - x\right) = \sin\left(\frac{\pi}{2} + x\right)$$ "meet in the middle"

$$LHS = \sin\left(\frac{\pi}{2} - x\right) \underset{\text{formula}}{\overset{\text{subtraction}}{=}} \sin\left(\frac{\pi}{2}\right)\cos(x) - \cos\left(\frac{\pi}{2}\right)\sin(x)$$

$$= 1 \cdot \cos(x) - 0 \cdot \sin(x)$$

$$= \cos(x)$$

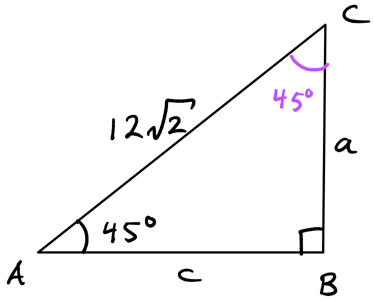
$$RHS = \sin\left(\frac{\pi}{2} + x\right) \underset{\text{formula}}{\overset{\text{addition}}{=}} \sin\left(\frac{\pi}{2}\right)\cos(x) + \cos\left(\frac{\pi}{2}\right)\sin(x)$$

$$= 1 \cdot \cos(x) + 0 \cdot \sin(x)$$

$$= \cos(x)$$

Since $LHS = RHS$ it is proven.

4. (a) A right triangle ABC has one acute angle 45°. The hypotenuse is length $12\sqrt{2}$. Solve the triangle.



For $\angle C$:

$\angle A + \angle B + \angle C = 180°$

$45° + 90° + \angle C = 180°$

$\angle C = 180° - 135° = \boxed{45°}$

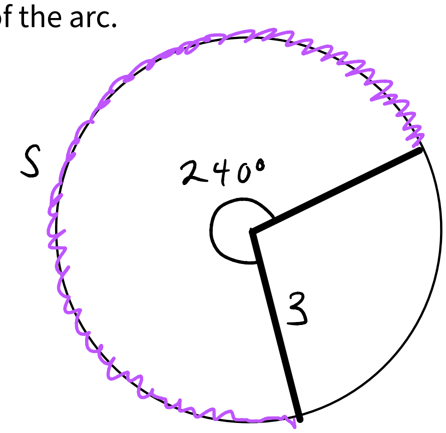
For a: $\sin(45°) = \frac{a}{12\sqrt{2}}$

$a = 12\sqrt{2}\sin(45°) = \cancel{12}^{6}\sqrt{2} \cdot \frac{\sqrt{2}}{\cancel{2}} = 6 \cdot \sqrt{2}\sqrt{2} = 6 \cdot 2 = \boxed{12}$

For c: same calculation as a. $\boxed{c = 12}$

You can also use the similar triangle technique with the 45°-45°-90° triangle.

(b) A central angle of 240° subtends an arc in a circle of radius 3 centimeters. Find the length of the arc.



needs to be in rad

$S = r \cdot \theta$

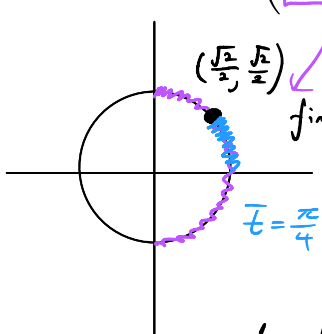
$= 3 \cdot \cancel{240°}^{4} \cdot \frac{\pi}{\cancel{180°}_{3}}$ rad

$= \cancel{3} \cdot \frac{4\pi}{\cancel{3}}$ rad

$= \boxed{4\pi \text{ rad}}$

(c) Evaluate $\tan\left(\underbrace{\sin^{-1}\frac{\sqrt{2}}{2}}\right)$.

find t value for y-coordinate in the shaded portion



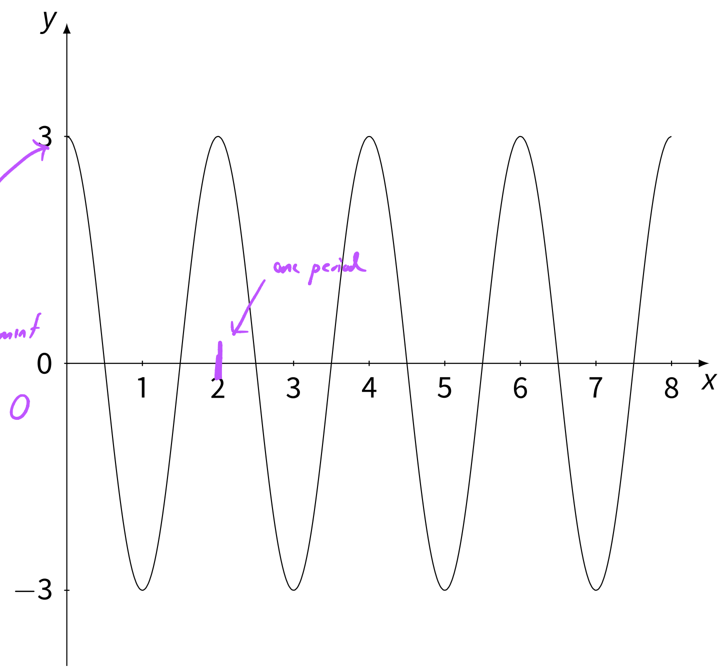
$\tan\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \tan\left(\frac{\pi}{4}\right)$

$= \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= \boxed{1}$

5. Suppose a mass attached to a spring is moving in simple harmonic motion. The displacement $f(t)$ is shown in the following graph.

must use cosine; displacement does not start at 0



Here, t is measured in seconds and $f(t)$ is measured in centimeters.

(a) Find a function $f(t)$ describing the displacement.

$f(t) = a \cos \omega t$

vertical stretch; $\boxed{3}$ from above graph

for ω: use period $= \frac{2\pi}{\omega}$

$2 = \frac{2\pi}{\omega}$

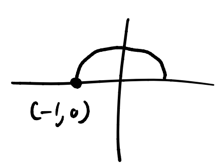
$\omega = \frac{\cancel{2}\pi}{\cancel{2}} = \boxed{\pi}$

$\boxed{f(t) = 3\cos(\pi t)}$

(b) How many centimeters is the mass displaced after one second?

$f(1) = 3\cos(\pi \cdot 1)$

$= 3\cos(\pi)$



$= 3 \cdot (-1)$

$= \boxed{-3 \text{ cm.}}$